Code Optimization

Code Optimization

- Requirements:
  - Meaning must be preserved (correctness)
  - Speedup must occur on average
  - Work done must be worth the effort
- Opportunities:
  - Programmer (algorithm, directives)
  - Intermediate code
  - Target code

Levels

- Window – peephole optimization
- Basic block
- Procedural – global (control flow graph)
- Program level – interprocedural

Peephole Optimizations

- Constant Folding

`x := 32` becomes `x := 64`

`x := x + 32`

- Unreachable Code

`goto L2`

`x := x + 1` ← unneeded

- Flow of control optimizations

`goto L1` becomes `goto L2`

…

`L1: goto L2`

Peephole Optimizations

- Algebraic Simplification

  `x := x + 0` ← unneeded

- Dead code

  `x := 32` ← where x not used after statement

  `y := x + y` → `y := y + 32`

- Reduction in strength

  `x := x * 2` → `x := x + x`

Peephole Optimizations

- Local in nature
- Pattern driven
- Limited by the size of the window

Basic Block Level

- Common Subexpression Elimination
- Constant Propagation
- Dead code elimination
- Plus many others such as copy propagation, value numbering, partial redundancy elimination, …

Simple example: a[i+1] = b[i+1]

t1 = i +1	t1 = i +1	
t2 = b[t1]	t2 = b[t1]	
t3 = i + 1	t3 = i + 1	← *no longer live*
a[t3] = t2	a[t1] = t2	

Common expression can be eliminated

Simple example: a[i+1] = b[i+1]

Now, suppose i is a constant:

i = 4	i = 4	i = 4
t1 = i +1	t1 = 5	t1 = 5
t2 = b[t1]	t2 = b[t1]	t2 = b[5]
a[t1] = t2	a[t1] = t2	a[5] = t2

Final code:

```
i = 4
t2 = b[5]
a[5] = t2
```

Control Flow Graph - CFG

CFG = < V, E, Entry >, where

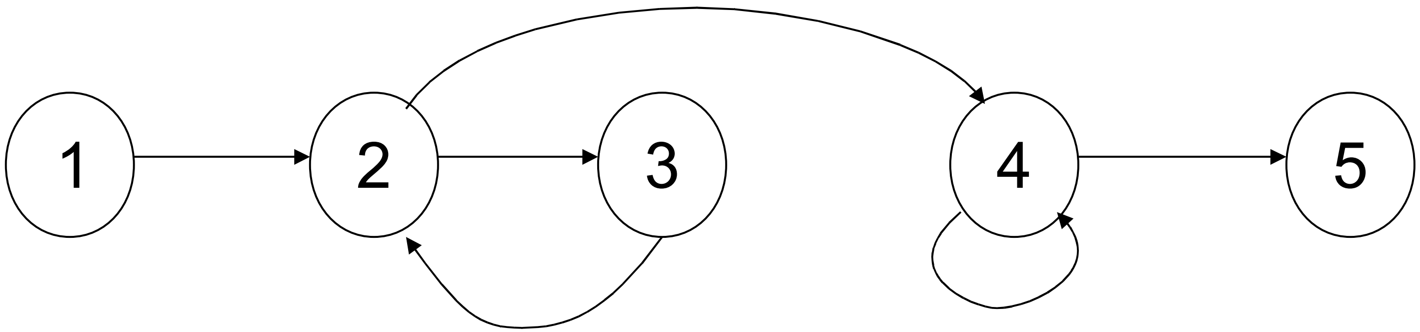
V = vertices or nodes, representing an instruction or basic block (group of statements).

E = (V x V) edges, potential flow of control

Entry is an element of V, the unique program entry

Two sets used in algorithms:

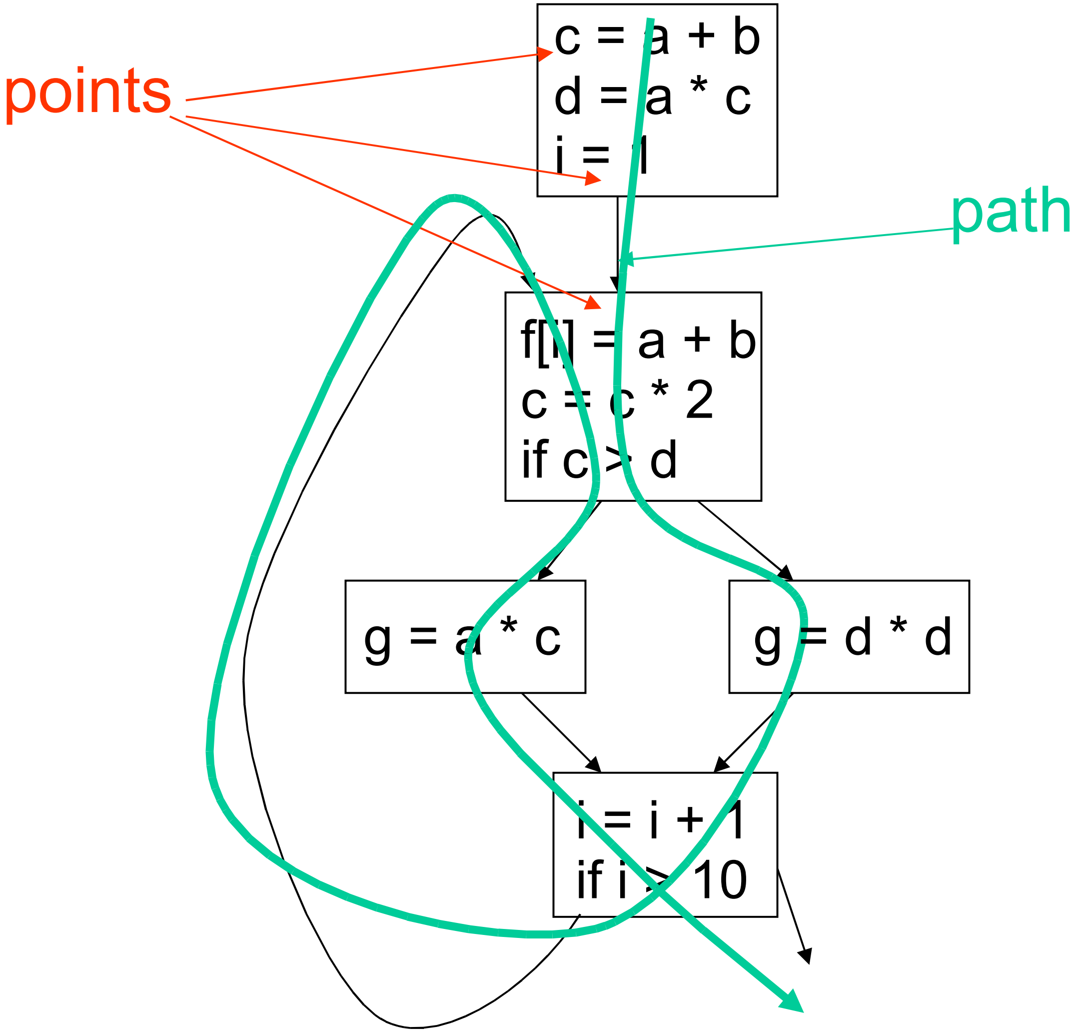
- Succ(v) = {x in V| exists e in E, e = v → x}
- Pred(v) = {x in V| exists e in E, e = x → v}



Definitions

- point - any location between adjacent statements and before and after a basic block.
- A path in a CFG from point p_1 to p_n is a sequence of points such that ∀ j, 1 <= j < n, either p_i is the point immediately preceding a statement and p_{i+1} is the point immediately following that statement in the same block, or p_i is the end of some block and p_{i+1} is the start of a successor block.

CFG



Optimizations on CFG

- Must take control flow into account
  - Common Sub-expression Elimination
  - Constant Propagation
  - Dead Code Elimination
  - Partial redundancy Elimination
  - …
- Applying one optimization may create opportunities for other optimizations.

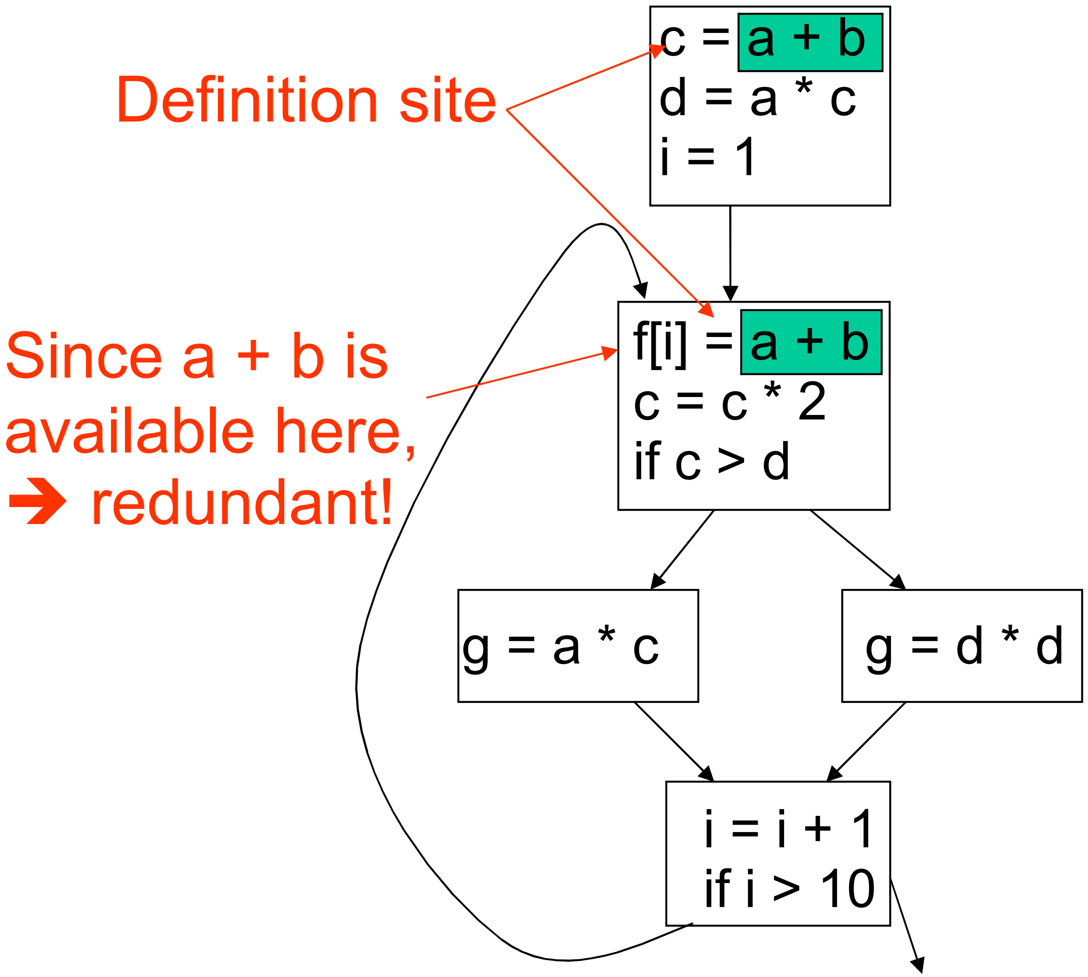
Redundant Expressions

An expression **`x op y`** is redundant at a point p if it has already been computed at some point(s) and no intervening operations redefine **`x`** or **`y`**.

```
m = 2*y*z          t0 = 2*y          t0 = 2*y
                   m = t0*z          m = t0*z
n = 3*y*z          t1 = 3*y          t1 = 3*y
                   n = t1*z          n = t1*z
o = 2*y-z          t2 = 2*y
                   o = t2-z          o = t0-z
```

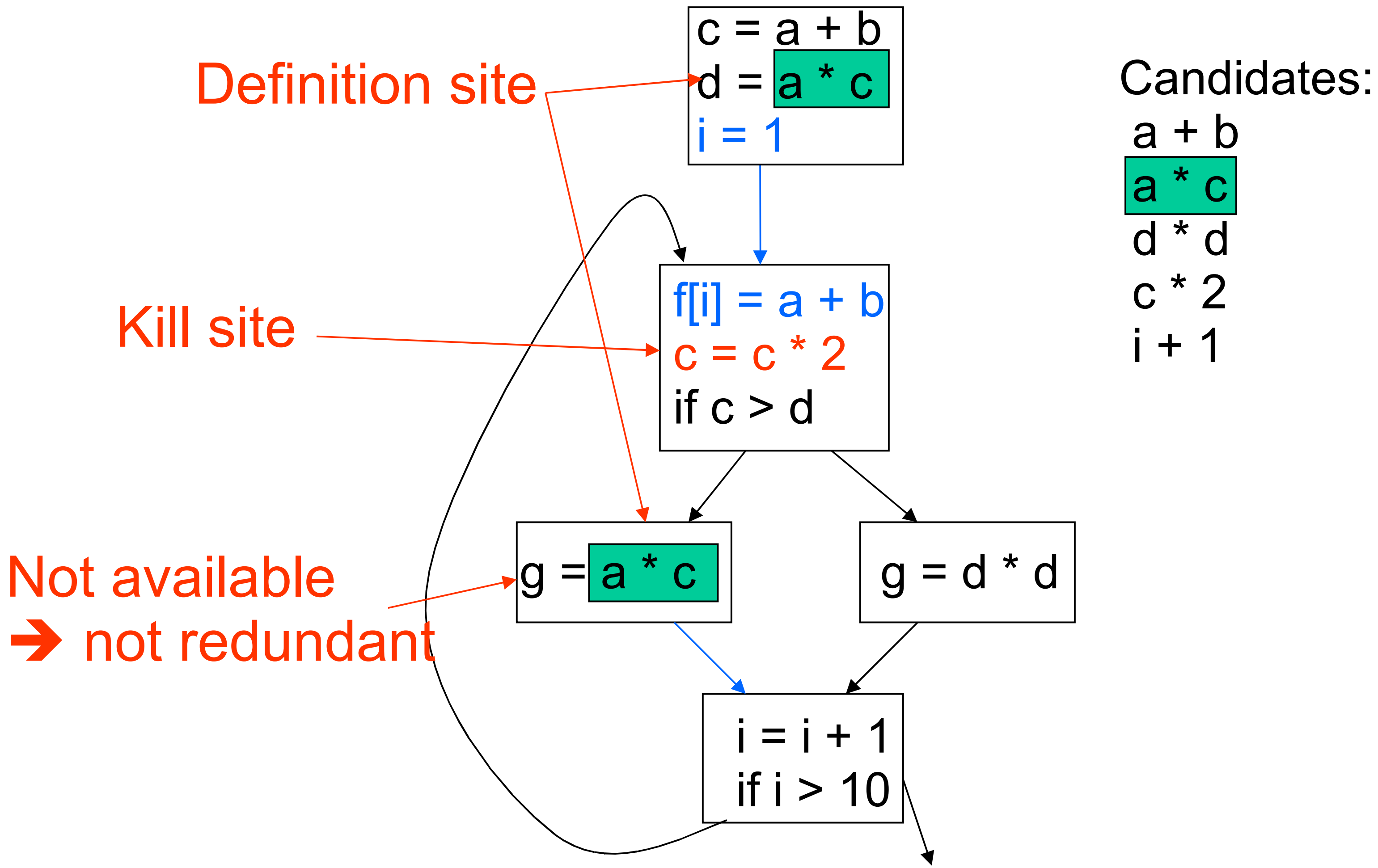
redundant

Redundant Expressions



Candidates:
a + b
a * c
d * d
c * 2
i + 1

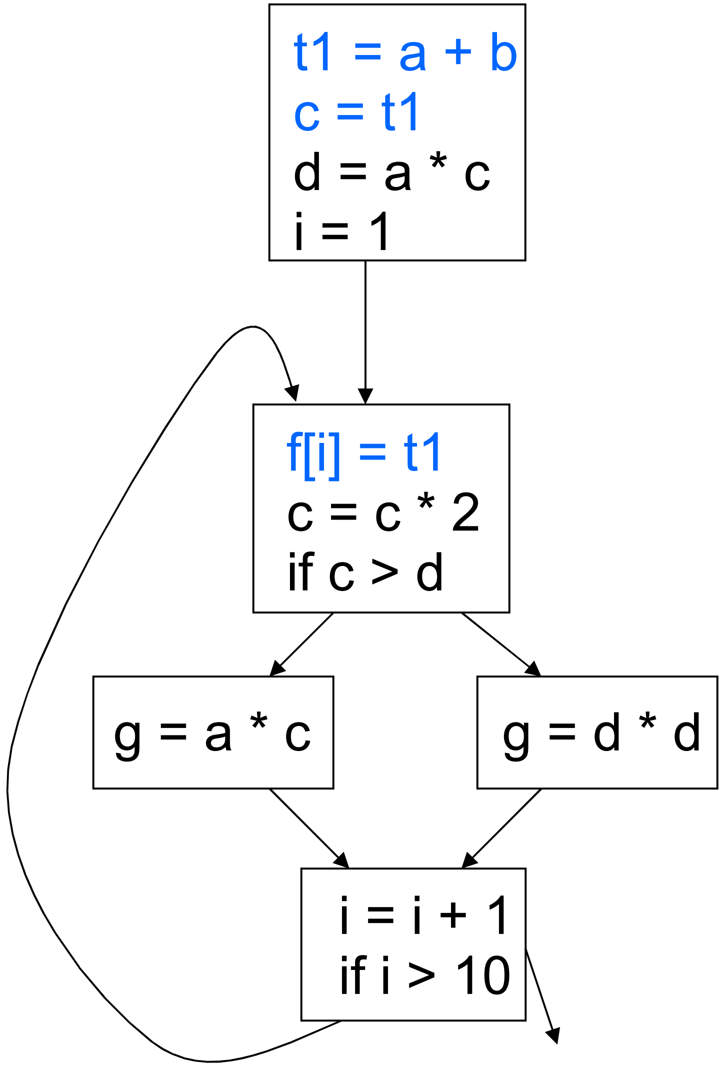
Redundant Expressions



Redundant Expressions

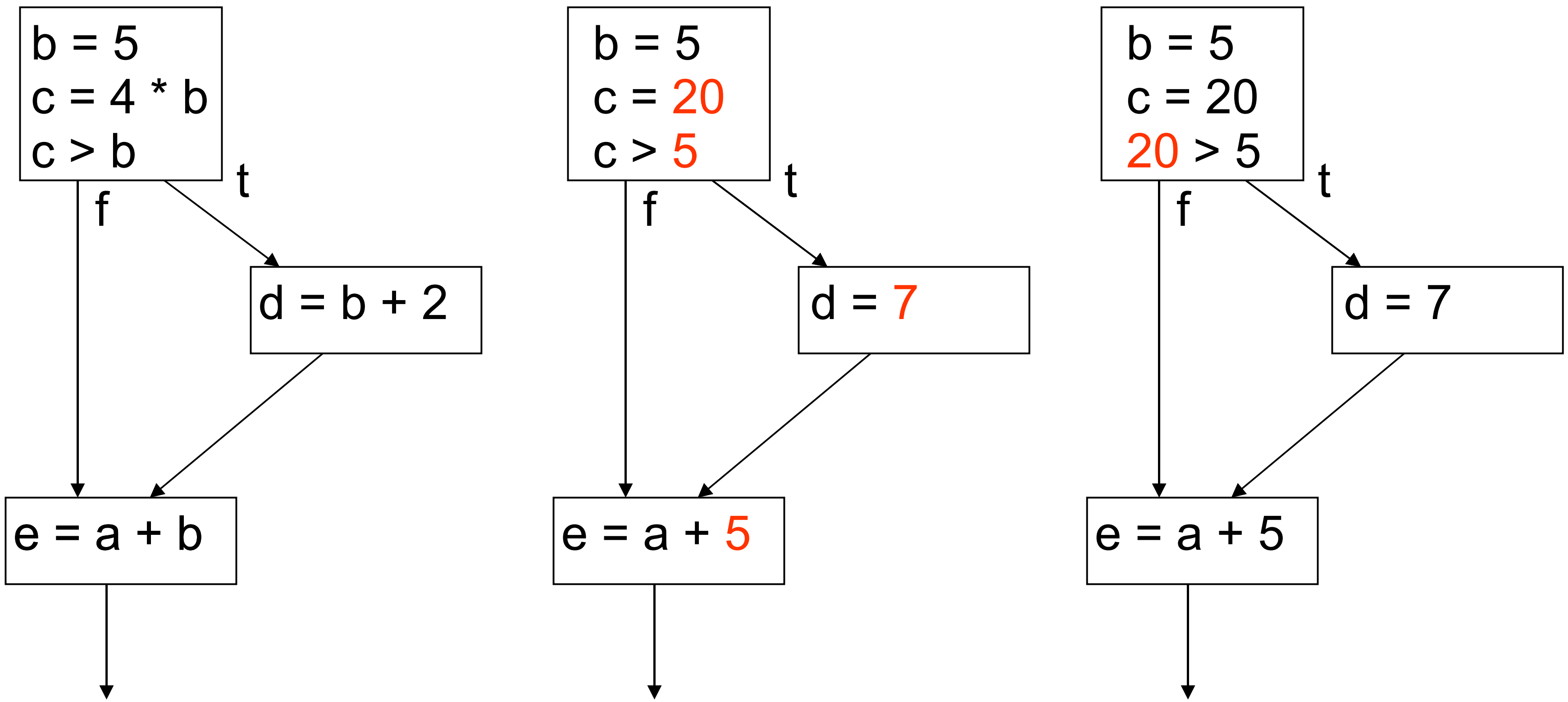
- An expression e is defined at some point p in the CFG if its value is computed at p. (definition site)
- An expression e is killed at point p in the CFG if one or more of its operands is defined at p. (kill site)
- An expression is available at point p in a CFG if every path leading to p contains a prior definition of e and e is not killed between that definition and p.

Removing Redundant Expressions

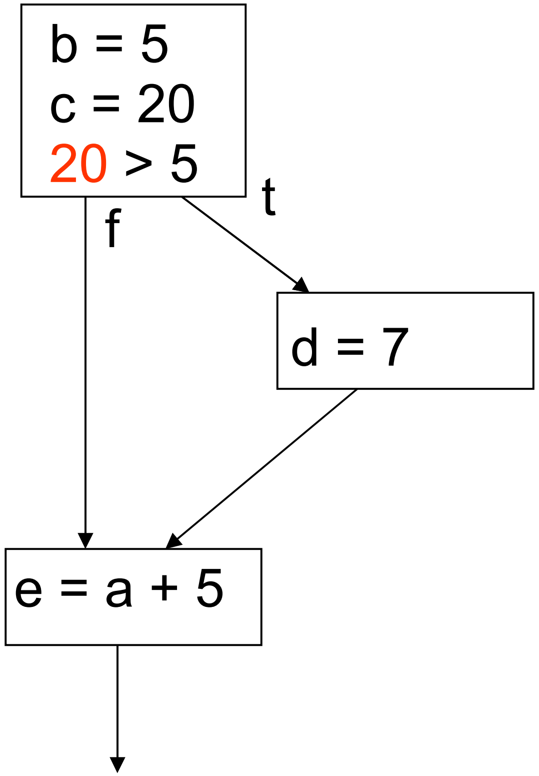


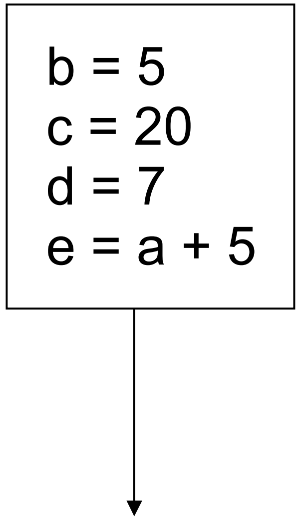
Candidates:
a + b
~~a * c~~
~~d * d~~
~~c * 2~~
~~i + 1~~

Constant Propagation

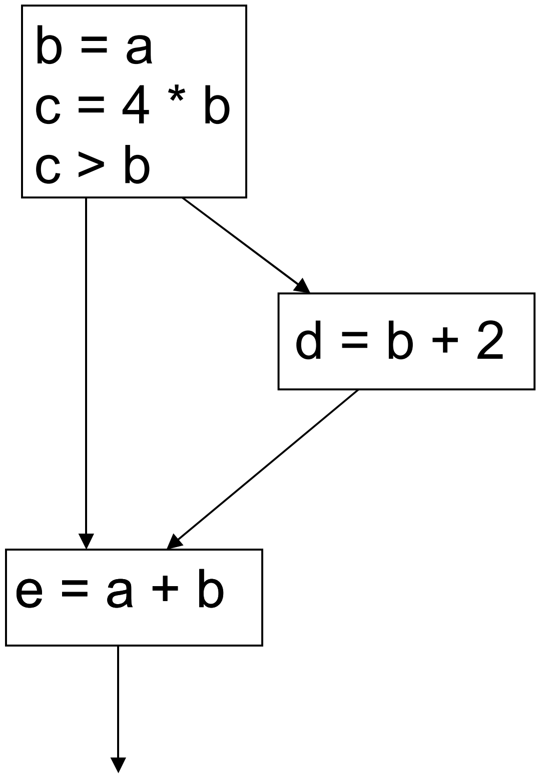


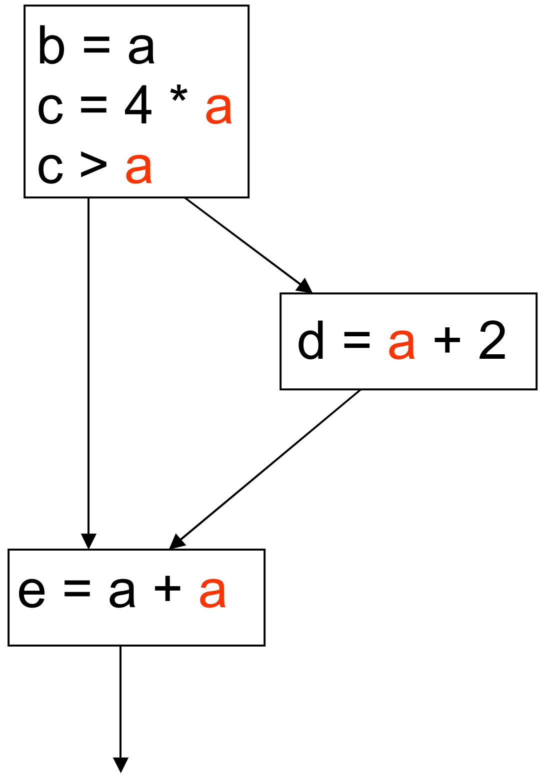
Constant Propagation





Copy Propagation





Simple Loop Optimizations: Code Motion

```
while (i <=  limit - 2)
```

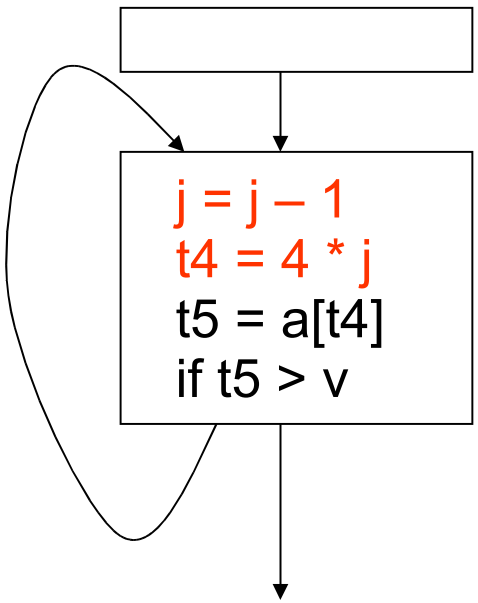
```
L1:
    t1 = limit - 2
    if (i > t1) goto L2
    body of loop
    goto L1
L2:
```

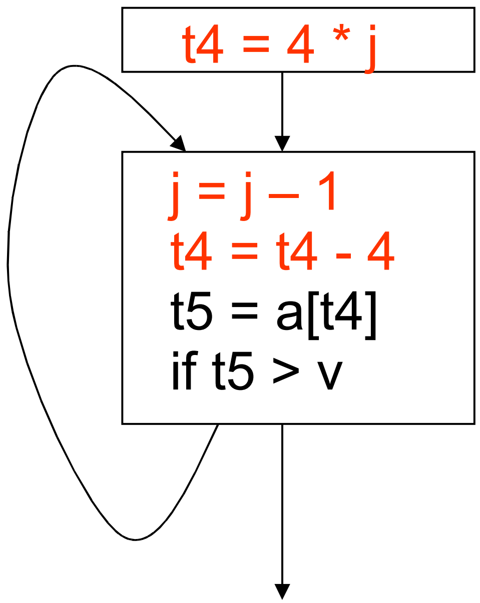
```
t := limit - 2
   while (i <= t)
```

```
    t1 = limit - 2
L1:
    if (i > t1) goto L2
    body of loop
    goto L1
L2:
```

Simple Loop Optimizations: Strength Reduction

- Induction Variables control loop iterations



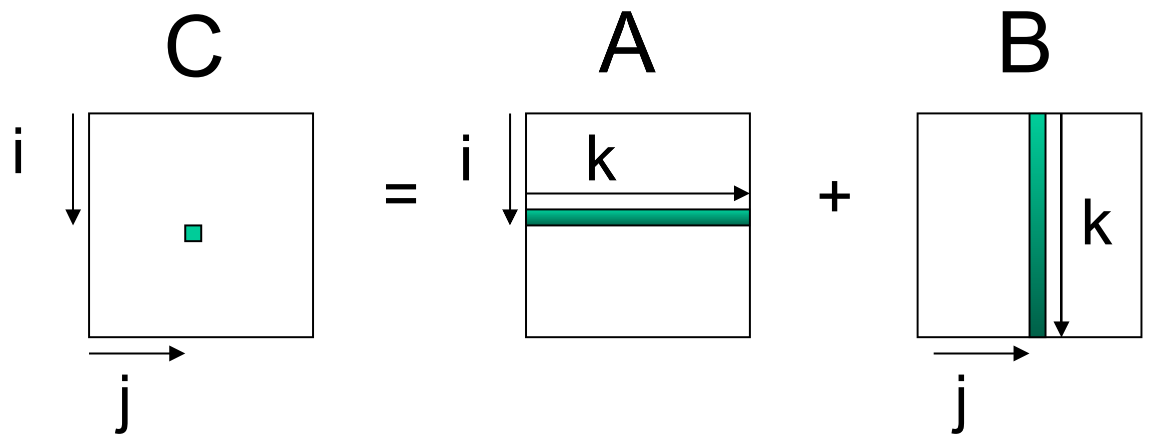


Simple Loop Optimizations

- Loop transformations are often used to expose other optimization opportunities:
  - Normalization
  - Loop Interchange
  - Loop Fusion
  - Loop Reversal
  - …

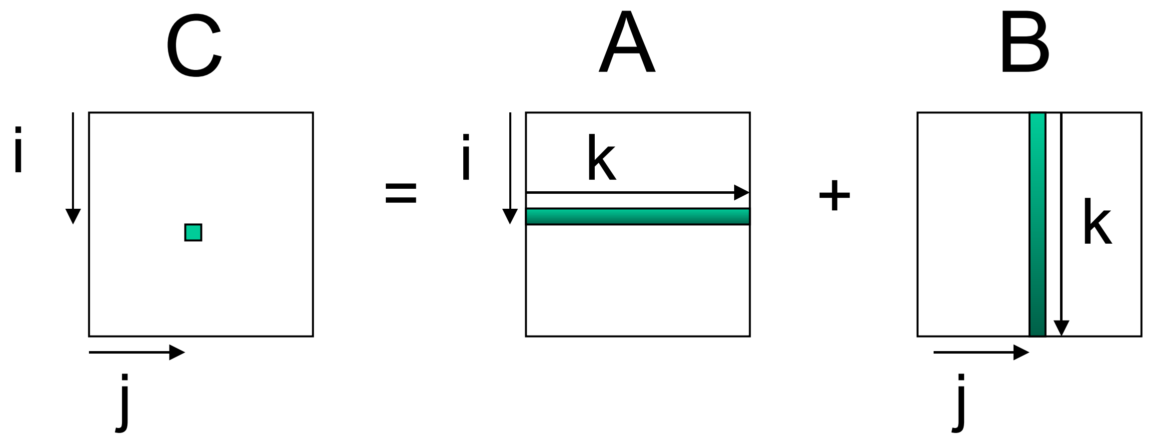
Consider Matrix Multiplication

```
for i = 1 to n do
  for j = 1 to n do
     for k = 1 to n do
        C[i,j] = C[i,j] + A[i,k] + B[k,j]
     end
  end
end
```



Memory Usage

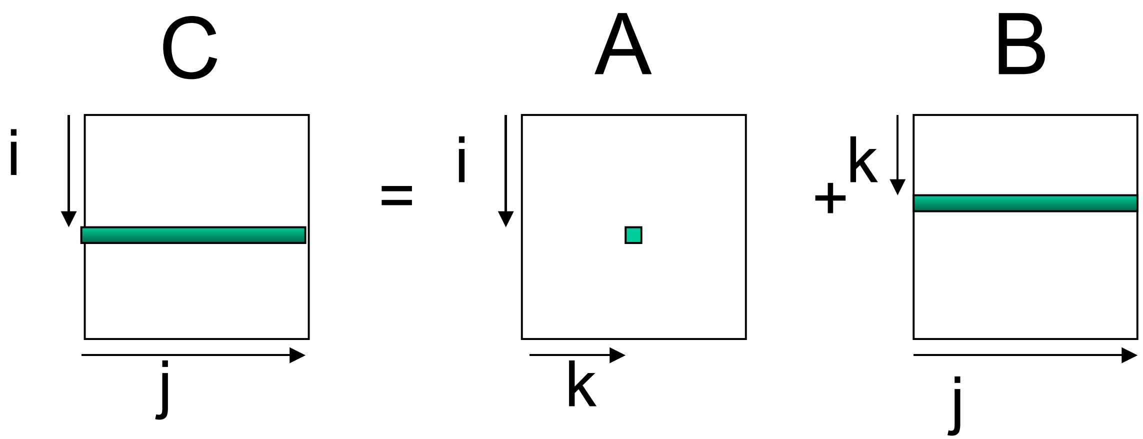
- **For A:** Elements are accessed across rows, spatial locality is exploited for cache (assuming row major storage)
- **For B:** Elements are accessed along columns, unless cache can hold all of B, cache will have problems.
- **For C:** Single element computed per loop – use register to hold



Matrix Multiplication - Version 2

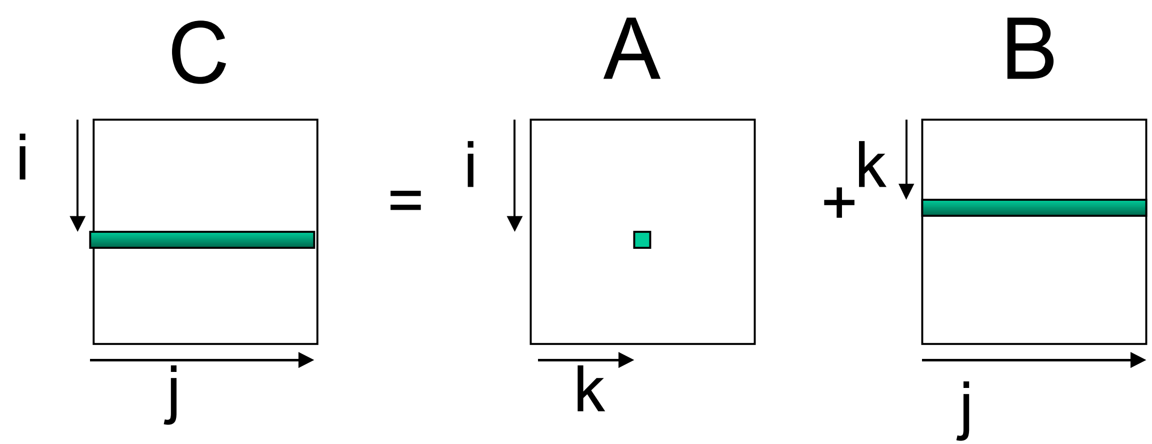
```
for i = 1 to n do
  for k = 1 to n do
     for j = 1 to n do
        C[i,j] = C[i,j] + A[i,k] + B[k,j]
     end
  end
end
```

loop interchange



Memory Usage

- **For A:** Single element loaded for loop body
- **For B:** Elements are accessed along rows to exploit spatial locality.
- **For C:** Extra loading/storing, but across rows



Simple Loop Optimizations

- How to determine safety?
  - Does the new multiply give the same answer?
  - Can be reversed?
  - `for (I=1 to N) a[I] = a[I+1]`
    - can this loop be safely reversed?

Data Dependencies

- Flow Dependencies - write/read

  x := 4;

  y := x + 1

- Output Dependencies - write/write

  x := 4;

  x := y + 1;

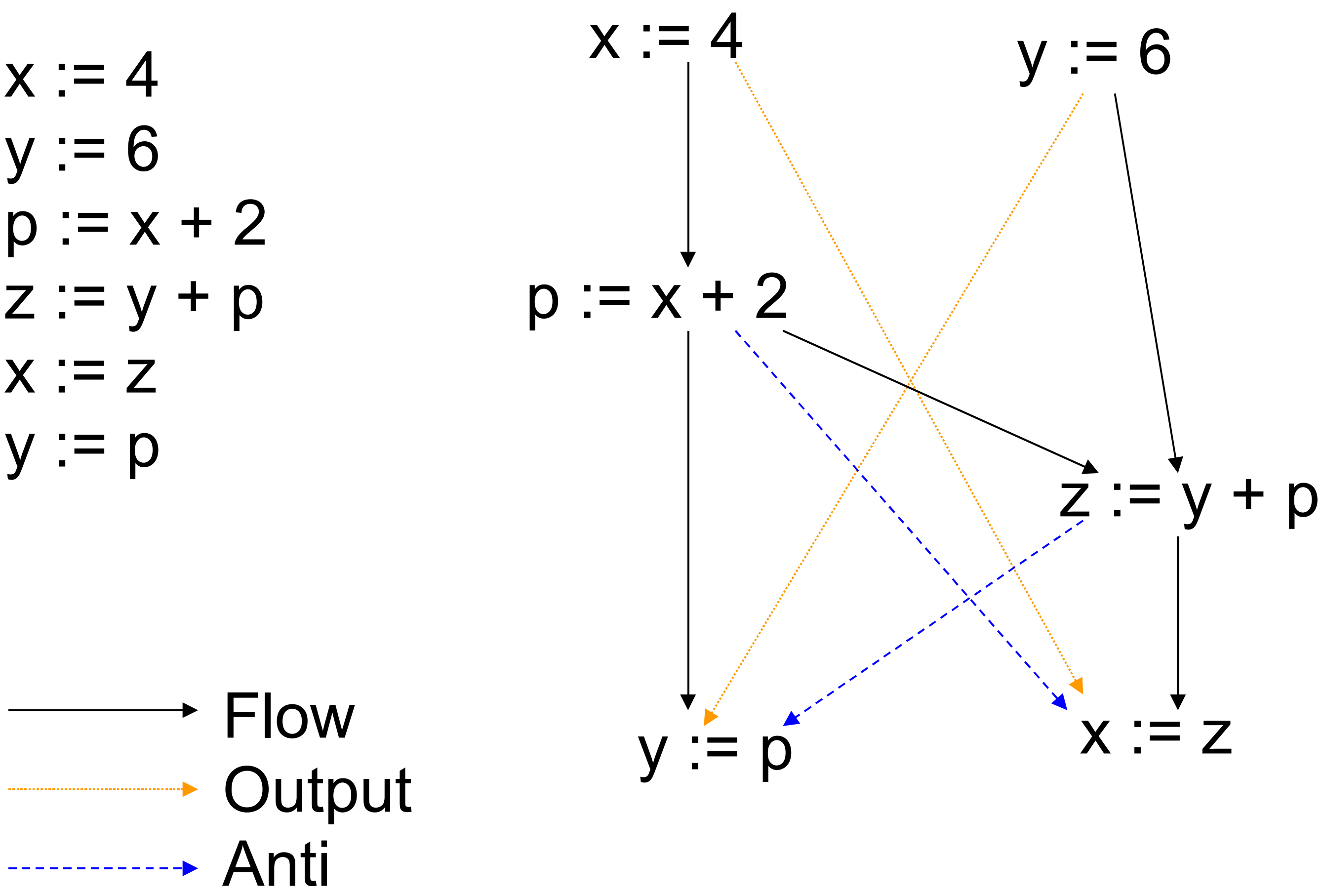
- Antidependencies - read/write

  y := x + 1;

  x := 4;

Data Dependencies - Example

x := 4
y := 6
p := x + 2
z := y + p
x := z
y := p



Global Data Flow Analysis

Collecting information about the way data is used in a program.

- Takes control flow into account
- HL control constructs
  - Simpler – syntax driven
  - Useful for data flow analysis of source code
- General control constructs – arbitrary branching
- Information needed for optimizations such as: constant propagation, common sub-expressions, partial redundancy elimination …

Dataflow Analysis: Iterative Techniques

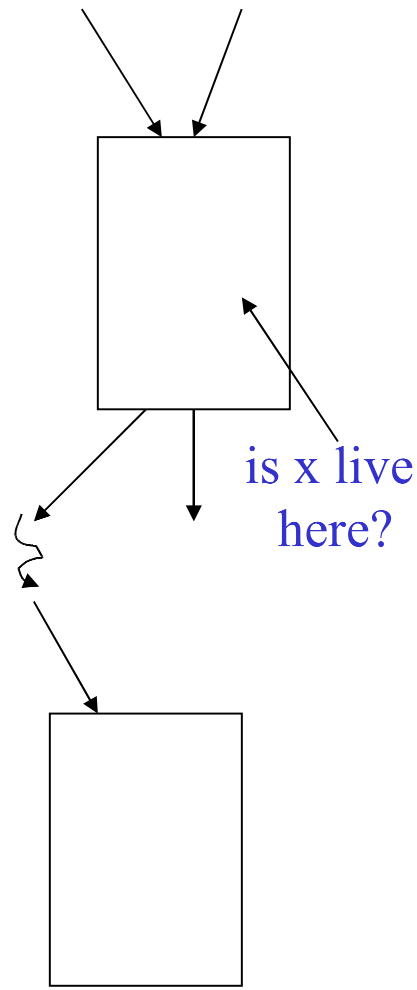
- First, compute local (block level) information
- Iterate until no changes

```
while change do
     change = false
     for each basic block
          apply equations updating IN and OUT
          if either IN or OUT changes, set change to true
end
```

Live Variable Analysis

- A variable **x** is *live* at a point p if there is some path from p where **x** is used before it is defined.
- Want to determine for some variable x and point p whether the value of x *could* be used along some path starting at p.

- Information flows backwards
- May – ‘along some path starting at p’



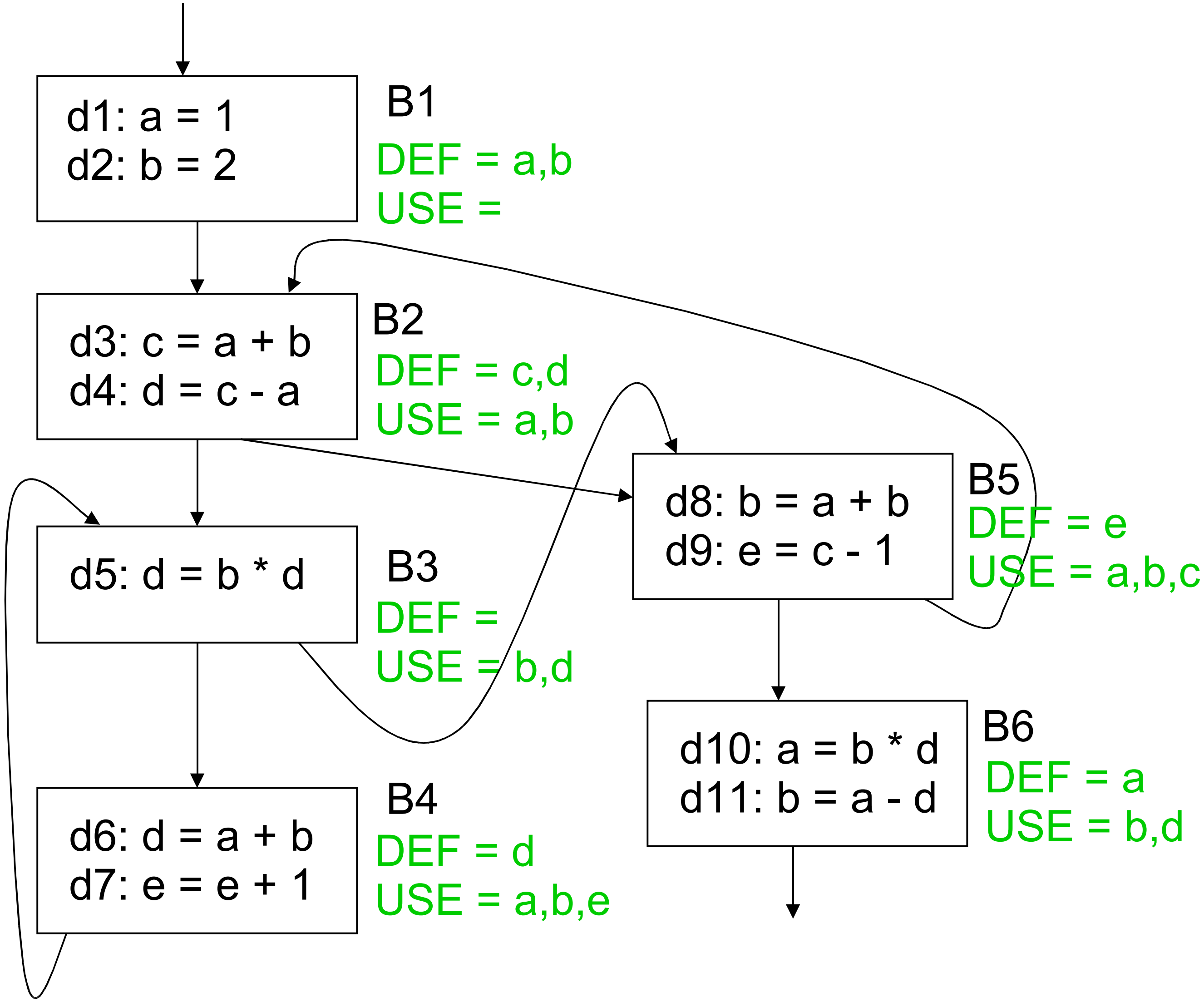
Global Live Variable Analysis

- Want to determine for some variable *x* and point *p* whether the value of *x* could be used along some path starting at *p*.

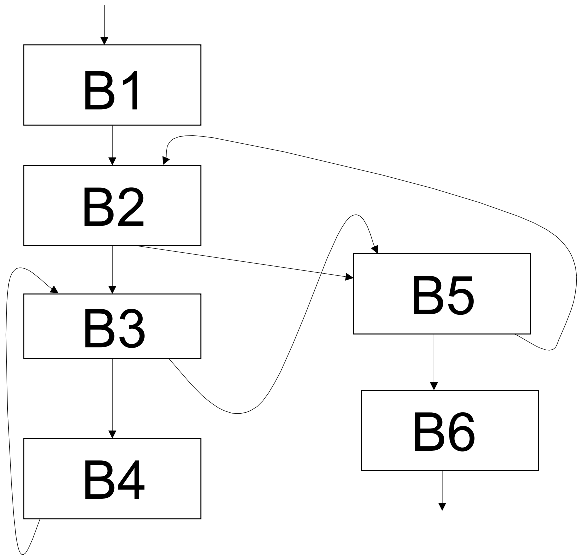
- DEF[B] - set of variables assigned values in B prior to any use of that variable
- USE[B] - set of variables used in B prior to any definition of that variable
- OUT[B] - variables live immediately after the block OUT[B]: $\cup$IN[S] for all S in succ(B)
- IN[B] - variables live immediately before the block

$$IN[B] = USE[B] + (OUT[B] - DEF[B])$$

DEF[B] and USE[B]



Example



Block	DEF	USE
B1	{a,b}	{ }
B2	{c,d}	{a,b}
B3	{ }	{b,d}
B4	{d}	{a,b,e}
B5	{e}	{a,b,c}
B6	{a}	{b,d}

OUT[B] = ∪ IN[S] for all S in succ(B)
IN[B] = USE[B] + (OUT[B] - DEF[B])

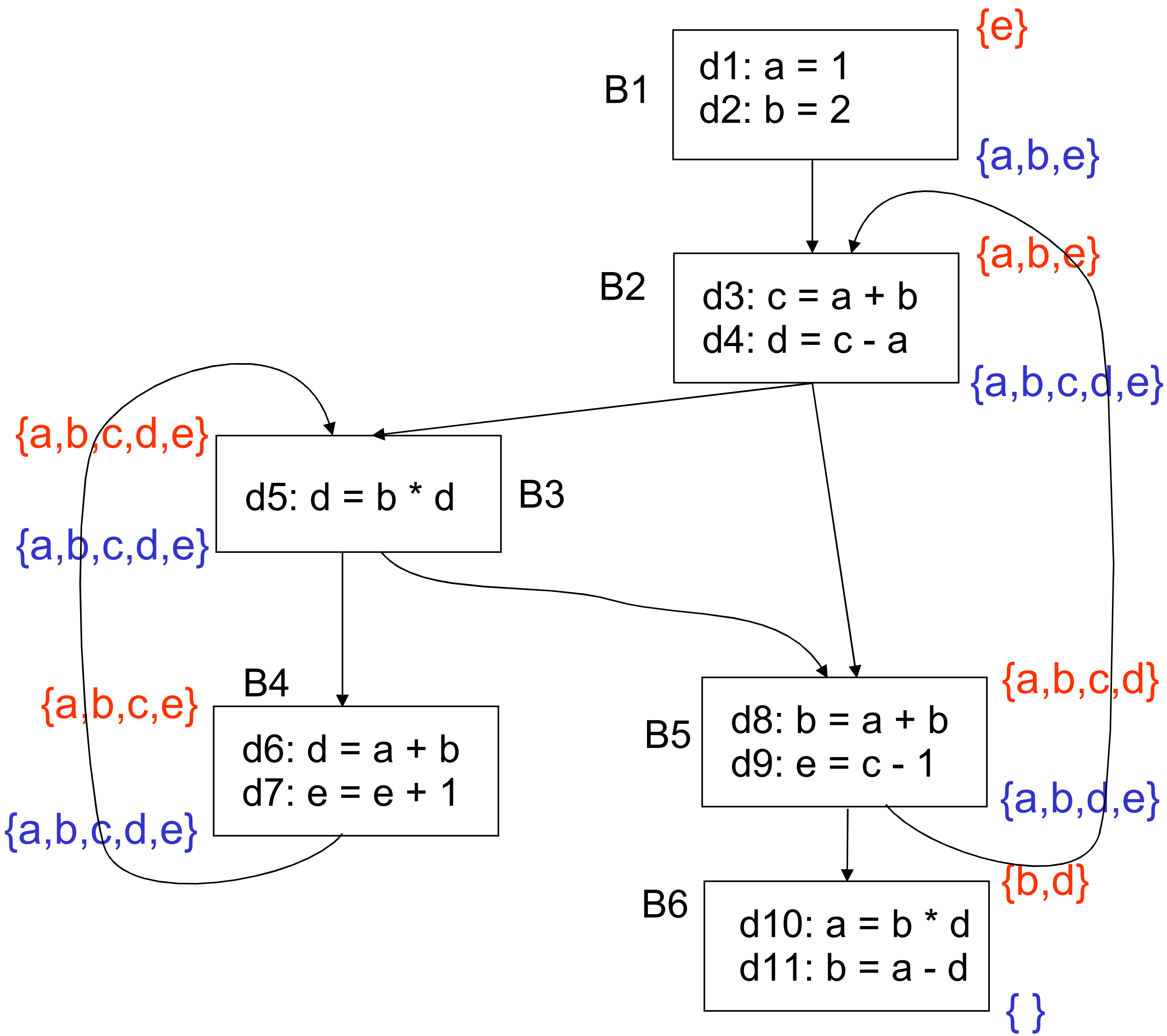
OUT[B1] = IN[B2]
IN[B1] = OUT[B1] - {a,b}
OUT[B2] = IN[B3] + IN[B5]
IN[B2] = {a,b} + (OUT[B2] - {c,d})
OUT[B3] = IN[B4] + IN[B5]
IN[B3] = {b,d} + OUT[B3]
OUT[B4] = IN[B3]
IN[B4] = {a,b,e} + (OUT[B4] - {d})
OUT[B5] = IN[B6] + IN[B2]
IN[B5] = {a,b,c} + (OUT[B5] - {e})
OUT[B6] = ∅
IN[B6] = {b,d} + (OUT[B6] - {a})

	IN	OUT
B1	∅	
B2	∅	
B3	∅	
B4	∅	
B5	∅	
B6	∅	

Solution

	IN	OUT	IN	OUT	IN	OUT	IN	OUT	IN
B1	∅	∅	∅	a,b	∅	a,b	∅	a,b,e	e
B2	∅	∅	a,b	a,b,c,d	a,b	a,b,c,d,e	a,b,e	a,b,c,d,e	a,b,e
B3	∅	∅	b,d	a,b,c,e	a,b,c,d,e	a,b,c,d,e	a,b,c,d,e	a,b,c,d,e	a,b,c,d,e
B4	∅	b,d	a,b,e	a,b,c,d,e	a,b,c,e	a,b,c,d,e	a,b,c,e	a,b,c,d,e	a,b,c,e
B5	∅	a,b	a,b,c	a,b,d	a,b,c,d	a,b,d,e	a,b,c,d	a,b,d,e	a,b,c,d
B6	∅	∅	b,d	∅	b,d	∅	b,d	∅	b,d

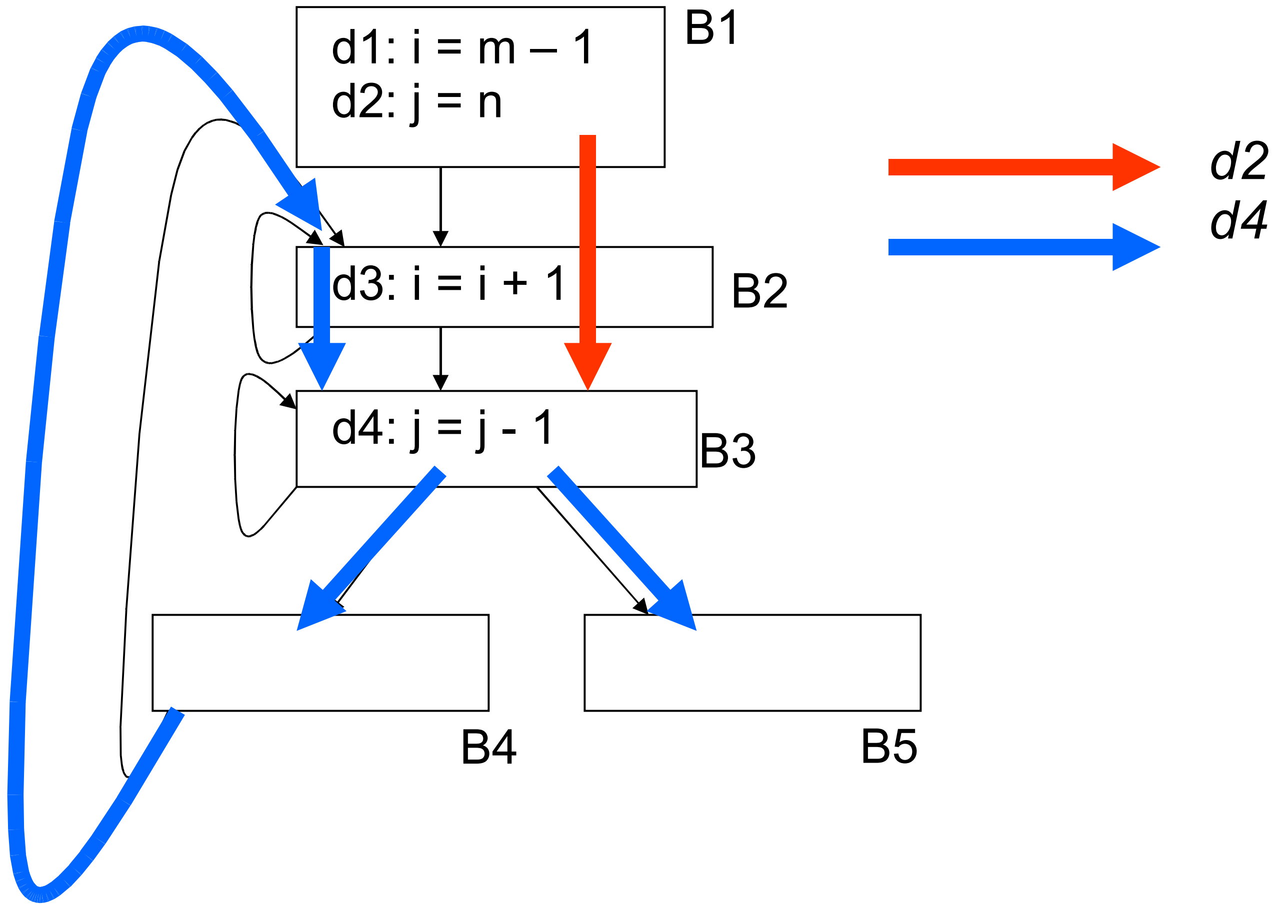
CFG after Live Variable Analysis



Dataflow Analysis Problem #2: Reachability

- A definition of a variable x is a statement that may assign a value to x
- A definition may reach a program point p if there exists some path from the point immediately following the definition to p such that the assignment is not killed along that path
- Concept: relationship between definitions and uses

What Blocks Do Definitions d2 and d4 Reach?



Reachability Analysis: Unstructured Input

1. Compute GEN and KILL at block-level
2. Compute IN[B] and OUT[B] for B

   IN[B] = U OUT[P]   *where P is a predecessor of B*

   OUT[B] = GEN[B] U (IN[B] - KILL[B])

3. Repeat step 2 until there are no changes to OUT sets

Reachability Analysis: Step 1

- For each block, compute local (block level) information = GEN/KILL sets
  - GEN[B] = set of definitions generated by B
  - KILL[B] = set of definitions that can not reach the end of B
- This information does not take control flow between blocks into account

Reasoning about Basic Blocks

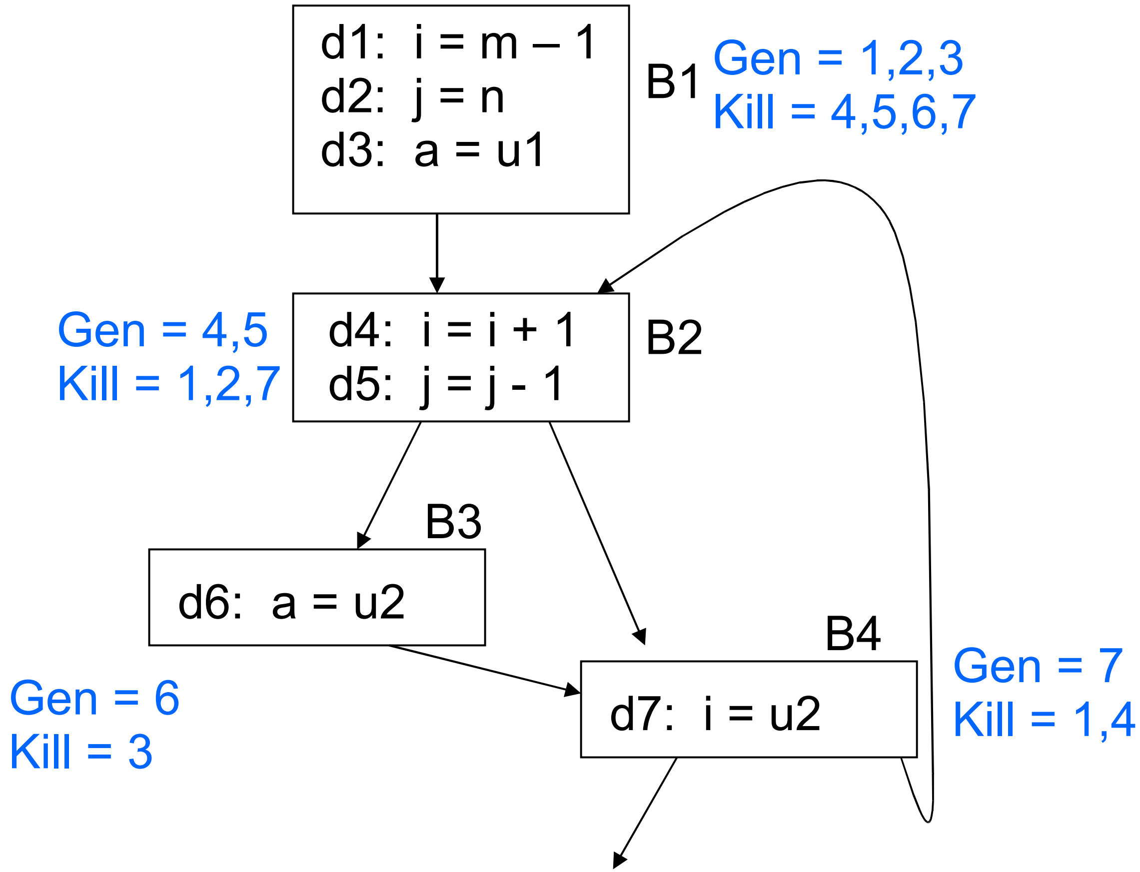
Effect of single statement: a = b + c

- Uses variables {b,c}
- **Kills all definitions of {a}**
- **Generates new definition (i.e. assigns a value) of {a}**

Local Analysis:

- Analyze the effect of each instruction
- Compose these effects to derive information about the entire block

Example



d1: i = m – 1
d2: j = n
d3: a = u1
B1
Gen = 1,2,3
Kill = 4,5,6,7
Gen = 4,5
Kill = 1,2,7
d4: i = i + 1
d5: j = j - 1
B2
B3
d6: a = u2
Gen = 6
Kill = 3
B4
d7: i = u2
Gen = 7
Kill = 1,4

Reachability Analysis: Step 2

- Compute IN/OUT for each block in a forward direction. Start with IN[B] = $\varnothing$
  - IN[B] = *set of defns reaching the start of B*

    = $\cup$ (out[P]) for all predecessor blocks in the CFG
  - OUT[B] = *set of defns reaching the end of B*

    = GEN[B] $\cup$ (IN[B] – KILL[B])

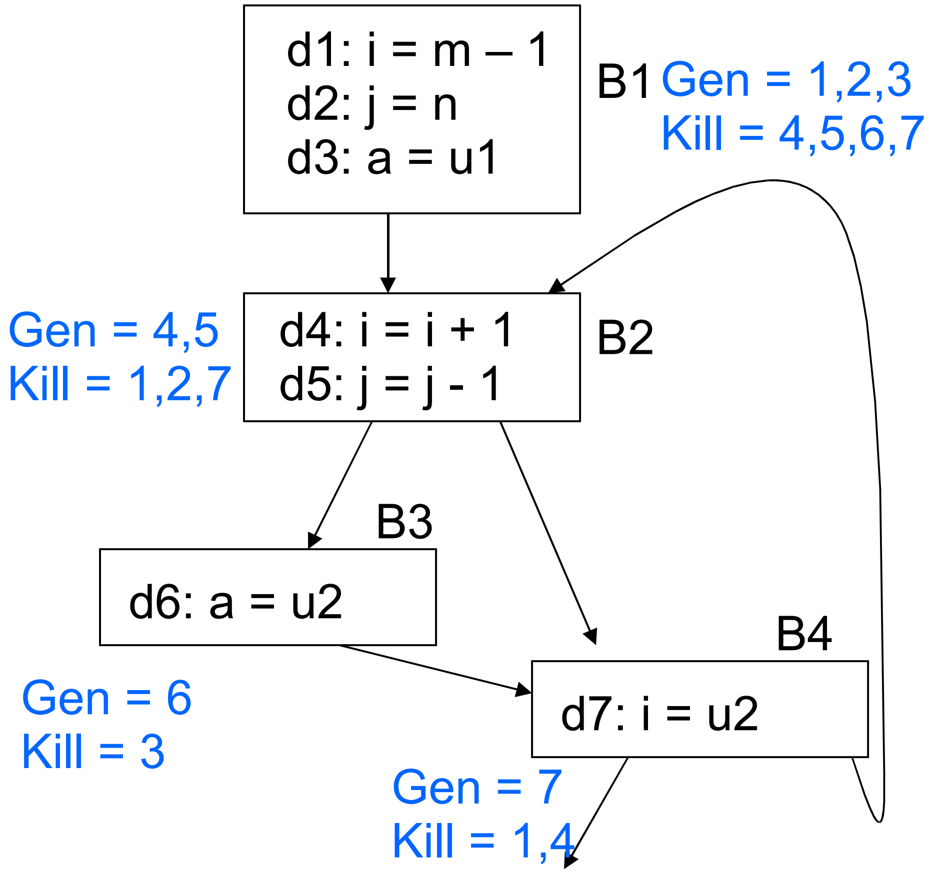
- Keep computing IN/OUT sets until a fixed point is reached

Reaching Definitions Algorithm

- Input: Flow graph with GEN and KILL for each block
- Output: in[B] and out[B] for each block.

```
For each block B do out[B] = gen[B], (true if in[B] = emptyset)
change := true;
while change do begin
        change := false;
        for each block B do begin
          in[B] := U out[P], where P is a predecessor of B;
          oldout = out[B];
          out[B] := gen[B] U (in[B] - kill [B])
          if out[B] != oldout then change := true;
    end
end
```

Reaching Definitions Example



	IN	OUT		
B1	∅	1,2,3		
B2	∅	4,5		
B3	∅	6		
B4	∅	7		

IN[B] = ∪(out[P]) for all predecessor blocks in the CFG

OUT[B] = GEN[B] ∪ (IN[B] – KILL[B])

Reaching Definitions Example

IN[B] = ∪(out[P]) for all predecessor blocks in the CFG
OUT[B] = GEN[B] + (IN[B] – KILL[B])

	IN	OUT	IN	OUT
B1	∅	1,2,3	∅	1,2,3
B2	∅	4,5	OUT[1]+OUT[4] = 1,2,3,7	4,5 + (1,2,3,7 – 1,2,7) = 3,4,5
B3	∅	6	OUT[2] = 3,4,5	6 + (3,4,5 – 3) = 4,5,6
B4	∅	7	OUT[2]+OUT[3] = 3,4,5,6	7 + (3,4,5,6 – 1,4) = 3,5,6,7

Reaching Definitions Example

IN[B] = $\cup$(out[P]) for all predecessor blocks in the CFG
OUT[B] = GEN[B] + (IN[B] – KILL[B])

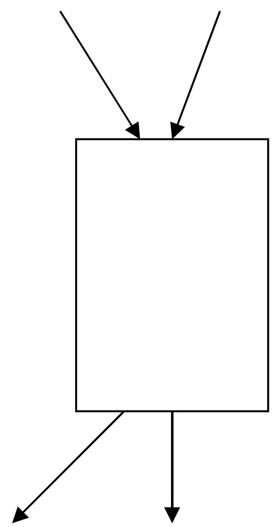
	IN	OUT	IN	OUT	IN	OUT
B1	∅	1,2,3	∅	1,2,3	∅	1,2,3
B2	∅	4,5	1,2,3,7	3,4,5	OUT[1] + OUT[4] = 1,2,3,5,6,7	4,5 + (1,2,3,5,6,7-1,2,7) = 3,4,5,6
B3	∅	6	3,4,5	4,5,6	OUT[2] = 3,4,5,6	6 + (3,4,5,6 – 3) = 4,5,6
B4	∅	7	3,4,5,6	3,5,6,7	OUT[2] + OUT[3] = 3,4,5,6	7+(3,4,5,6 – 1,4) = 3,5,6,7

Forward vs. Backward

- Forward flow vs. Backward flow
  - Forward: Compute OUT for given IN,GEN,KILL
    - Information propagates from the predecessors of a vertex
    - Examples: Reachability, available expressions, constant propagation
  - Backward: Compute IN for given OUT,GEN,KILL
    - Information propagates from the successors of a vertex
    - Example: Live variable analysis

Forward vs. Backward Equations

- Forward vs. backward
  - Forward:
    - IN[B] - process OUT[P] for all P in predecessors(B)
    - OUT[B] = local U (IN[B] – local)
  - Backward:
    - OUT[B] - process IN[S] for all S in successor(B)
    - IN[B] = local U (OUT[B] – local)

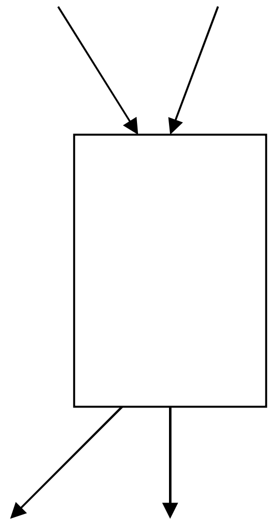


May vs. Must

- May vs. Must
  - Must – true on **all paths**
    - Ex: constant propagation – variable must provably hold appropriate constant on all paths in order to do a substitution
  - May – true on **some path**
    - Ex: Live variable analysis – a variable is live if it could be used on some path; reachability – a definition reaches a point if it can reach it on some path

May vs. Must Equations

- May vs. Must
  - May – IN[B] = $\cup$(out[P]) for all P in pred(B)
  - Must – IN[B] = $\cap$(out[P]) for all P in pred(B)



Example Equations

- Reachability
  - IN[B] = $\cup$(out[P]) for all P in pred(B)
  - OUT[B] = GEN[B] + (IN[B] – KILL[B])
- Live Variable Analysis
  - OUT[B] = $\cup$(IN[S]) for all S in succ(B)
  - IN[B] = USE[B] $\cup$ (OUT[B] - DEF[B])
- Constant Propagation
  - IN[B] = $\cap$(out[P]) for all P in pred(B)
  - OUT[B] = DEF_CONST[B] $\cup$ (IN[B] – KILL_CONST[B])

Discussion

- Why does this work?
  - Finite set – can be represented as bit vectors
  - Theory of lattices
- Is this guaranteed to terminate?
  - Sets only grow and since finite in size …
- Can we find ways to reduce the number of iterations?

Choosing Visit Order for Dataflow Analysis

- In forward flow analysis situations, if we visit the blocks in depth first order, we can reduce the number of iterations
- Suppose definition d follows block path 3 → 5 → 19 → 35 → 16 → 23 → 45 → 4 → 10 → 17 where the block numbering corresponds to the preorder depth-first numbering
- Then we can compute the reach of this definition in 3 iterations of our algorithm

3 → 5 → 19 → 35 → 16 → 23 → 45 → 4 → 10 → 17